HW7 solution

7.2.12

(a) As in Example 7.2.1, we have that the posterior distribution of μ is given by the

$$N\left(\left(1+\frac{10}{9}\right)^{-1}\left(65+\left(\frac{10}{9}\right)63.20\right),\left(1+\frac{10}{9}\right)^{-1}\right)=N\left(64.053,\frac{9}{19}\right).$$

The posterior mode is then $\hat{\mu}=64.053$. A .95-credible interval for μ is given by $64.053\pm\sqrt{9/19}z_{0.975}=(62.704,65.402)$. Since this interval has length equal to 2.698 and the margin of error is less then 1.5 marks (which is quite small) we conclude that the estimate is quite accurate.

(b) Based on the .95-credible interval, we cannot reject $H_0:\mu=65$, at the 5% level since 65 falls inside the interval.

(c) The posterior probability of the null hypothesis above is given by

$$\begin{aligned}\Pi(\mu=65\,|\,x_1,..,x_n)=&\frac{0.5m_1(s)}{0.5m_1(s)+0.5m_2(s)}\Pi_1(\mu=65\,|\,x_1,..,x_n)+\\&\frac{0.5m_1(s)}{0.5m_1(s)+0.5m_{21}(s)}\Pi_2(\mu=65\,|\,x_1,..,x_n)\end{aligned}$$

where $\Pi_2(\cdot\,|\,x_1,..,x_n)$ is as given in part (a) and $\Pi_1(\cdot\,|\,x_1,..,x_n)$ is degenerate at $\mu=65$.

The prior predictive under Π_1 is given by

$$\begin{aligned}m_1(x_1,..,x_n)&=(18\pi)^{-5}\exp\left(-\frac{(10-1)\,252.622}{(2)\,9}\right)\exp\left(-\frac{10}{18}(63.20-65)^2\right)\\&=3.981\times10^{-65}\end{aligned}$$

while the prior predictive under Π_2 is given by

$$\begin{aligned}m_2(x_1,..,x_n)=&(18\pi)^{-5}\exp\left(-\frac{(10-1)\,252.622}{(2)\,9}\right)\times\\&\exp\left(\frac{1}{2}\frac{9}{19}(135.22)^2\right)\exp\left(-\frac{1}{2}8663.0\right)(.68825)\\&=6.2662\times10^{-65}\end{aligned}$$

The posterior probability of the null is then equal to

$$\frac{0.5m_1(s)}{0.5m_1(s)+0.5m_2(s)}=\frac{3.981\times10^{-65}}{3.981\times10^{-65}+6.2662\times10^{-65}}=.3885.$$

(d) The Bayes factor in favor of $H_0 : \mu = 65$ is given by

$$BF_{H_0} = \frac{\exp\left(-\frac{10}{18}(63.20 - 65)^2\right)}{\exp\left(\frac{1}{2}\frac{9}{19}(135.22)^2\right)\exp\left(-\frac{1}{2}8663.0\right) \times .68825} = .6353.$$

7.2.15

(a) The odds in favor of A is defined by $P(A)/P(A^c)$. Hence,

$$\frac{P(A)}{P(A^c)} = \frac{P(A)}{1 - P(A)} = \frac{1 - P(A^c)}{P(A^c)} = 1\Big/\frac{P(A^c)}{1 - P(A^c)} = 1/\text{odds in favor of } A^c.$$

(b) The Bayes factor in favor of A is given by $BF(A) =$ posterior odds of A / prior odds of A.

$$BF(A) = \frac{\Pi(A|s)}{\Pi(A^c|s)}\Big/\frac{\Pi(A)}{\Pi(A^c)} = 1\Big/\left[\frac{\Pi(A^c|s)}{1 - \Pi(A^c|s)}\Big/\frac{\Pi(A)}{1 - \Pi(A^c)}\right] = 1/BF(A^c).$$

7.2.16 The fact that the odds of A is 3 implies $P(A)/(1 - P(A)) = 3$. This implies that $P(A) = 3/4$. If $\Pi(A) = 1/2$, then the prior odds of A is $\Pi(A)/\Pi(A^c) = (1/2)/(1/2) = 1$. The Bayes factor in favor of A is $BF(A) =$ posterior odds of A/ prior odds of $A = (\Pi(A|s)/(1 - \Pi(A|s)))/1 = 10$. This implies that $\Pi(A|s) = 10/11$.

7.2.18 Note that a credible set is an acceptance region and the compliment of γ-credible set is a $(1 - \gamma)$ rejection region. Since $\psi(\theta) = 0 \in (-3.3, 2.6)$, the P-value must be greater than $1 - 0.95 = 0.05$.